Progression in Calculation

St Peter's
Church of England (Aided) Primary School

A whole-school approach to teaching written methods

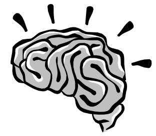
Contents 1

Contents 2

“They didn’t do it like that in my day!”

Which is a more appropriate method when calculating?:

mental calculation ?



or

written ?

Can I do this in my head?

Do I know the approximate size of the answer?

If I can’t do it wholly in my head, what do I need to write down in order to help me calculate the answer?

Will the written method I know be helpful?

This booklet has been produced to help with explaining modern methods of approaching maths.

Mental calculations

An ability to calculate mentally lies at the heart of numeracy. It is important to emphasise mental methods from the early years onwards, with regular opportunities to develop the different skills involved. Even when using written methods for calculations, we use our ability to calculate mentally.

These skills include:

- remembering number facts and recalling them without hesitation;
- using the facts that are known by heart to figure out new facts: for example, a fact like 8 + 6 = 14 can be used to work out 80 + 60 = 140, or 28 + 6 = 34;
- understanding and using the relationships between the 'four rules' to work out answers and check results: for example, 24 ÷ 4 = 6, since 6 x 4 = 24;
- drawing on a repertoire of mental strategies to work out calculations like 81 – 26, 23 x 4 or 5% of £3000, with some thinking time;
- solving problems like: 'Can I buy three bags of crisps at 35p each with my £1 coin?' or: 'Roughly how long will it take me to go 50 miles at 30 m.p.h.?

When do children need to start recording?

It is important to encourage children to look first at the problem and then get them to decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording or calculator.

The idea behind building towards formal written methods, as opposed to teaching formal methods straight away, is to secure children's understanding of our number system and how the operations function.

Children are encouraged to make informal jottings to deal with tricky mental calculations, and are then led to number line work, before moving on to more formal methods of recording.

Addition +

- add
- total
- and
- plus
- increase
- together
- sum
- more

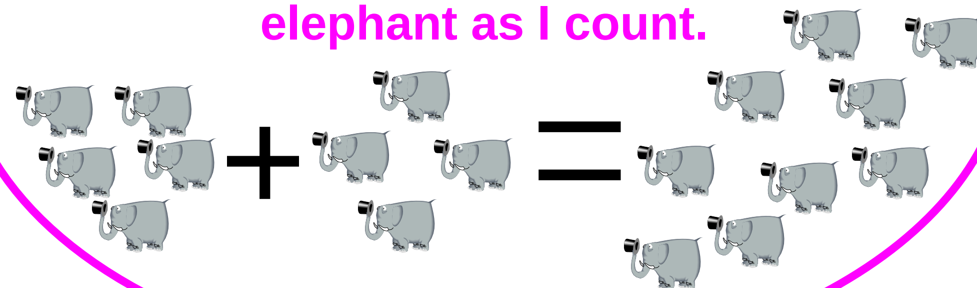
ADDITION

Adding using real objects.

U + U

5 + 4

It helps me to understand the question! I can then count the total while pointing to each elephant as I count.



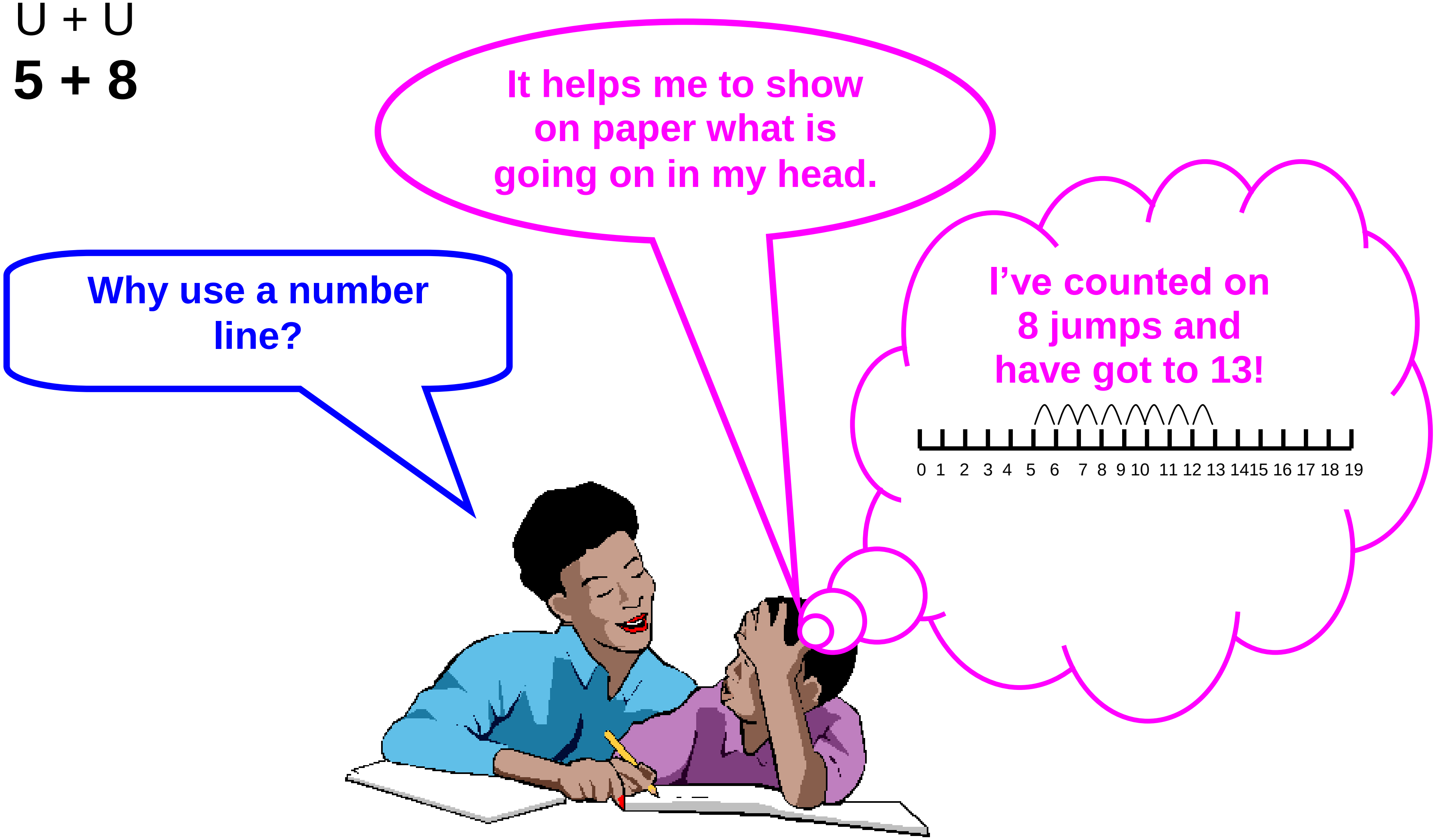
Why use toy elephants instead of doing it in your head?

ADDITION

Using a number line for adding single digit numbers.

U + U

5 + 8



ADDITION

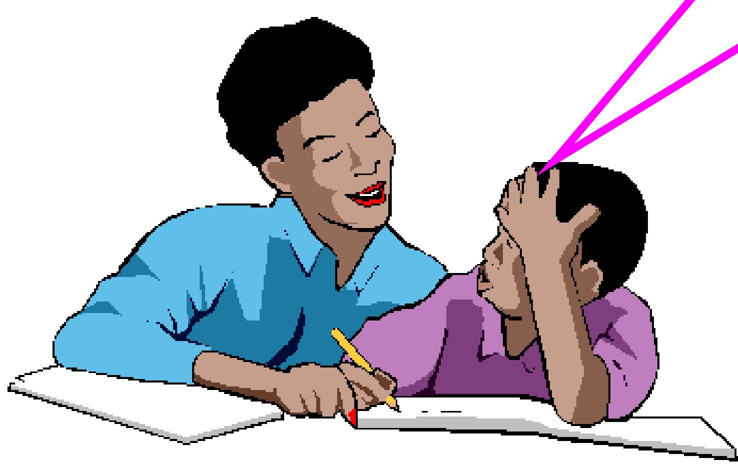
Using an informal method by counting on in multiples of 10 with a number line

TU + TU

86 + 57

Why use a number line?

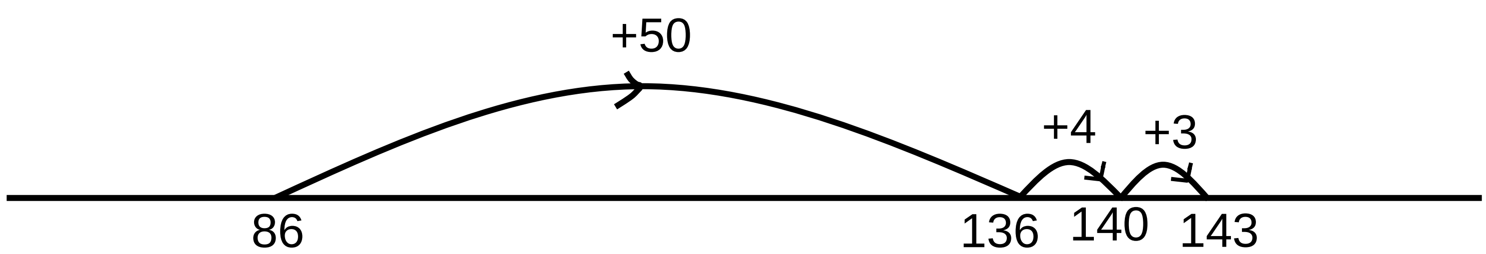
It helps me to show on paper what is going on in my head.



TU + TU

86 + 57

Start at 86 (the larger number) on the number line. *Partition* the smaller number 57 into tens and units and count on the multiples of 10 first and then the units.



86 + 57 = 143

ADDITION

Using a number line to add too much and then subtract (*compensate*)

HTU + TU

754 + 96

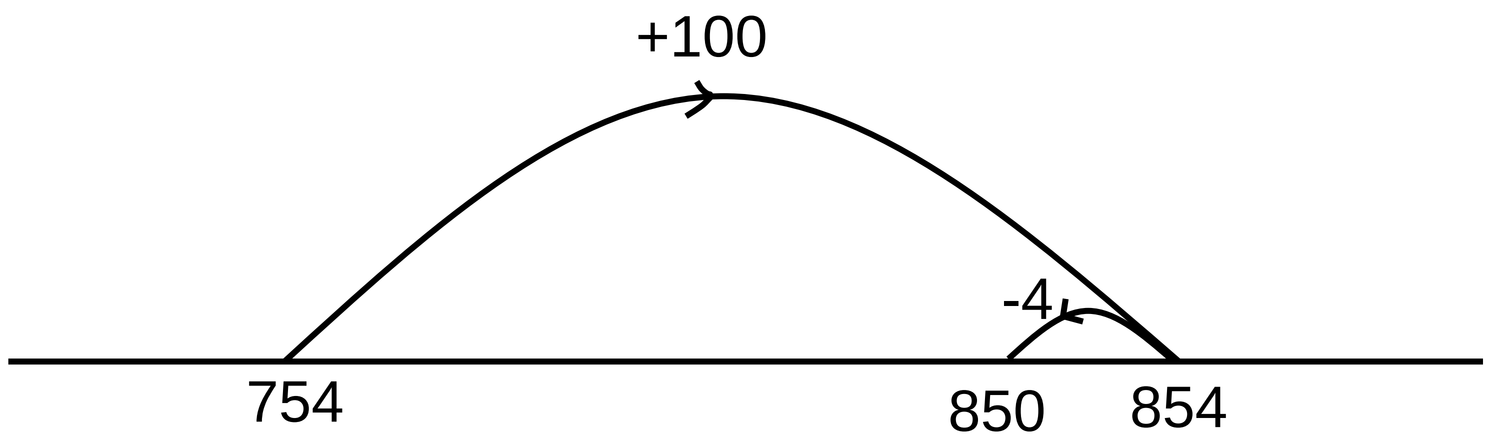
Why are you subtracting when you should be adding?

I noticed that 96 is close to 100. 100 is easier to add than 96 but that means I've added 4 too many. I need to subtract 4 from the number I reach.

HTU + TU

754 + 96

Start with the larger number 754. Add on 100 and then subtract 4.



754 + 96 = 850

ADDITION

HTU + TU

625 + 148

Expanded method: moving on from adding the *most significant digits* first to adding *least significant digits* first

Why switch to adding the units *(least significant digits)* first?

I know that I can add numbers in any order and the total will be the same. My teacher has told me that I need to practise adding the units first. The next method I will learn works this way. I must remember to line the numbers up in the correct columns.

HTU + HTU

625 + 148

Expanded method 1

Add *most significant digits* first:
(in this example, **hundreds**)

625	
+ 148	
700	600 + 100
60	20 + 40
13	5 + 8
773	

Add *least significant digits* first:
(in this example, **units**)

625	
+ 148	
13	5 + 8
60	20 + 40
700	600 + 100
773	

Mentally add
700 + 60 + 13 = 773

625 + 148 = 773

Expanded method 2

625 + 344 =

Partition and then add . . .

600 + 20 + 5
+ 300 + 40 + 4

900 + 60 + 9

= 969

ADDITION

Using a standard (compact) method

HTU + HTU

587 + 475

Why do you say 80 + 70 instead of 8 + 7?

I need to remember the value of each digit, so I know the size of the numbers I am adding and whether my answer makes sense.

HTU + HTU

587 + 475

7 + 5 = 12
Place the 2 in the units column and carry the 10 forward to the tens column.

	587
+	**475**
	1062
	1 1

80 + 70 = 150 then + 10 (carried forward) which totals 160.
Place 60 in the tens column and carry the 100 forward to the hundreds column.

500 + 400 = 900 then + 100 which totals 1000. Place this in the thousands column.

587 + 475 = 1062

Subtraction -

- difference between
- subtract
- fewer
- minus
- decrease take from
- reduce take away

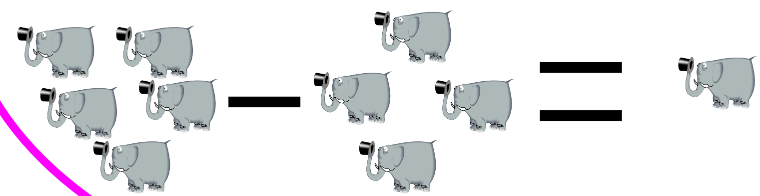
SUBTRACTION

Subtraction using real objects.

U - U

5 - 4

It helps me to understand the question! In this example I take four elephants away and there is only one left!



Why use toy elephants instead of doing it in your head?

SUBTRACTION

Using a number line for subtracting single digit numbers.

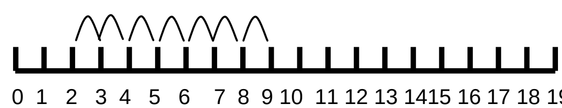
U - U

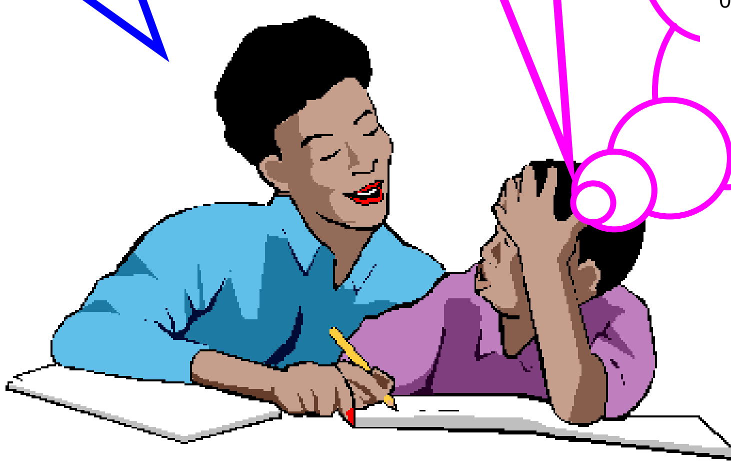
9 - 7

It helps me to show on paper what is going on in my head.

Why use a number line?

I've counted back 7 jumps and have got to 2!





SUBTRACTION

Ⓐ Counting on or Ⓑ counting back?

TU - TU

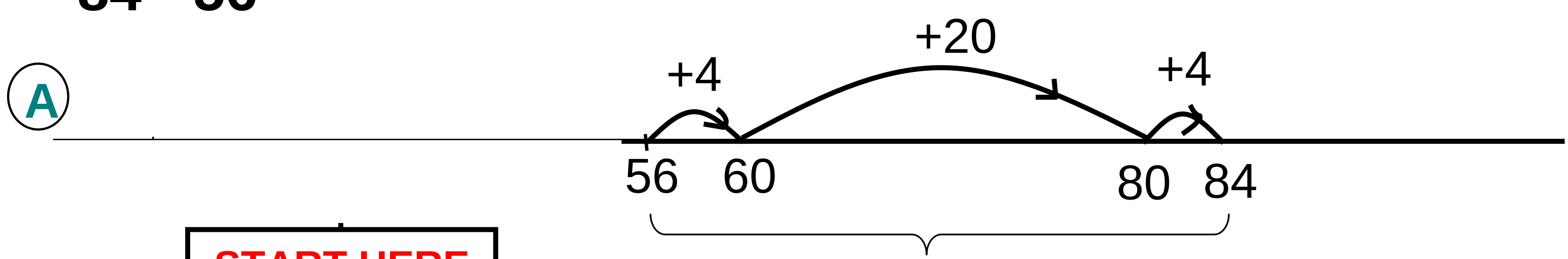
84 - 56

How do you decide whether to count on or count back?

If the numbers are close together like 203 – 198 it's quicker to count on. If they're a long way apart like 203 – 5 it's quicker to take away. Sometimes I count on because that's easier than taking away.

TU - TU

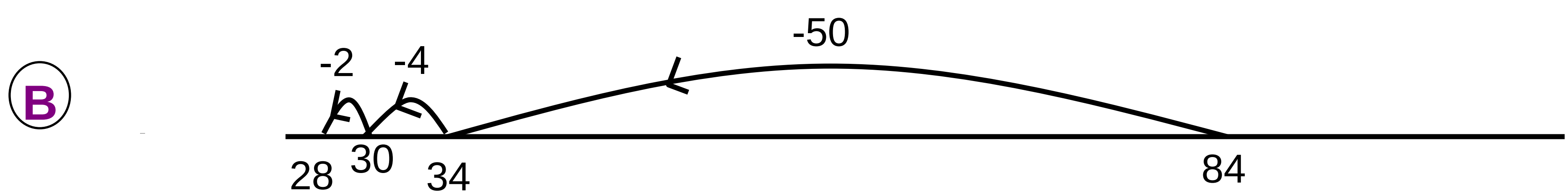
84 - 56



START HERE

Start by ‘taking away’ (crossing out) the 56.

Find the *difference* between the two numbers.
Count on from 56 to 84.
20 + 4 + 4 = 28



START HERE

Partition 56 and count back (subtract) 50 and then 6.

84 – 56 = 28

SUBTRACTION

HTU - HTU

954 - 586

Complementary addition

(A) Number line

(B) Written method

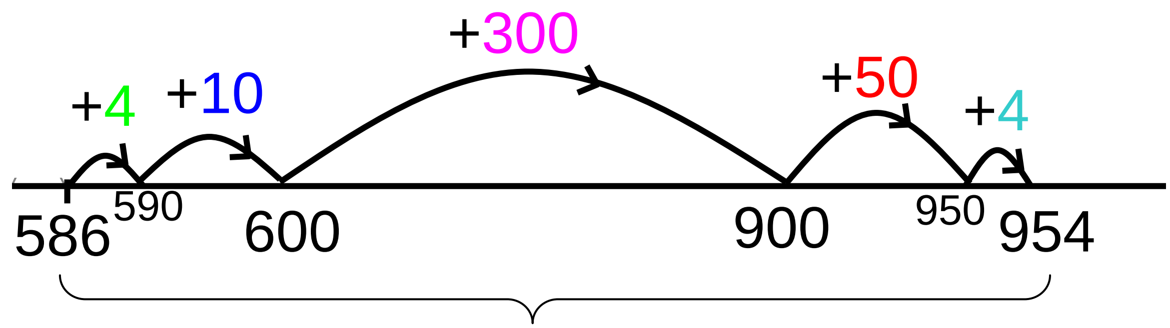
The number line method is very clear. Why do you use method B and write the numbers vertically?

I could make mistakes. Method B helps me line the numbers up and see what I need to add.

HTU - HTU

954 - 586

A



START HERE

‘Take away’ the 586.

Find the *difference* between the two numbers.
Count on from 586 to 954.
300 + 50 + 10 + 4 + 4 = 368

B

START HERE

- Count on to the next multiple of 10.
- Count on to the next multiple of 100.
- Count on in 100s.
- Count on to the larger number in the calculation which is 954.

954	
- 586	
4	**To make 590**
10	**To make 600**
300	**To make 900**
50	**To make 950**
4	**To make 954**
368	

954 - 586 = 368

SUBTRACTION

Working towards a standard (compact) method (*decomposition*)

HTU - TU

154 - 37

Why do you need to rearrange the numbers 50 + 4 and rewrite them as 40 + 14?

The whole number is 154. It is possible to subtract 7 but for this method I need to do one subtraction in each column. So I exchange one ten from the tens column for ten ones in the units column.

HTU - TU

154 - 37

Both these numbers are partitioned into their HTU parts, so we can do 3 easier calculations.

100 + 50 + 4
\- 30 + 7

54 is the same value as 40 10 4.
Now 7 can be subtracted from 14.

100 + ~~50~~ (40) + 4 (10)
\- 30 + 7

100 + 40 + 14
\- 30 + 7

100 + 10 + 7 = 117

Subtract the units, tens, then hundreds.

100 – 0 = 100

40 – 30 = 10

14 – 7 = 7

Here the answers from each calculation are added to give the answer.

154 - 37 = 117

SUBTRACTION

Standard (compact) method
(decomposition)

HTU - HTU

754 - 286

Why didn't you use the standard method straight away?

Because all the stages I have learnt before have really helped me understand exactly what I'm doing.

HTU - HTU

754 - 286

54 is the same value as 40 10 4.
Now 6 can be subtracted from 14.

740 is the same value as 600 + 100 + 40.
Now 80 can be subtracted from 140.

Or, more efficiently the *standard method.*

	700	+	~~50~~ (40)	+	(1) 4	
-	200	+	80	+	6	

	~~700~~ (600)	+	(1) 40	+	14	
-	200	+	80	+	6	

	600	+	140	+	14			
-	200	+	80	+	6			
	400	+	60	+	8	=	468	

```
 6 1 4 1
 ~~7~~~~5~~4
-286
----
 468
```

754 – 286 = 468

Multiplication x

- multiplied by
- multiply product
- groups of
- lots of
- times table
- times

MULTIPLICATION

Introducing multiplication in an array.

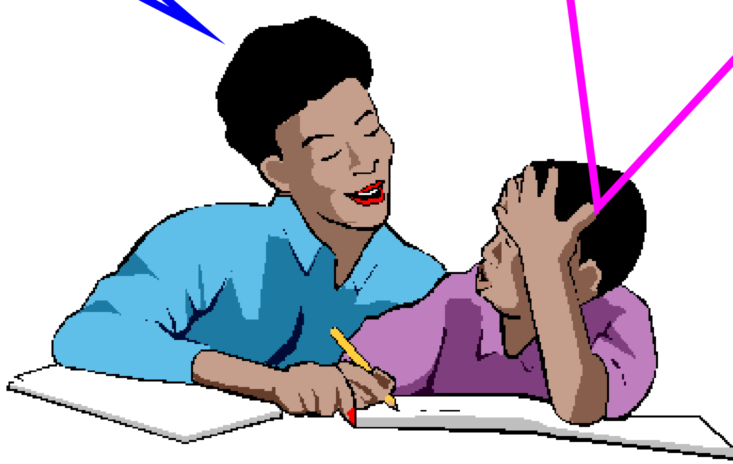
TU X U

3 x 5

How is multiplication the same as repeated addition?

The array helps me see how many lots of a number there are. Let's say there were 3 buses with 5 people on each.

I can count 15 people!



MULTIPLICATION

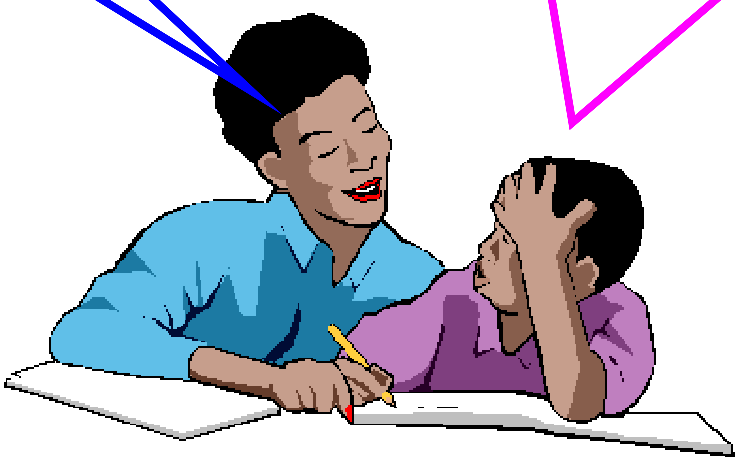
Introducing multiplication on a number line

TU X U

14 x 5

How is multiplication the same as repeated addition?

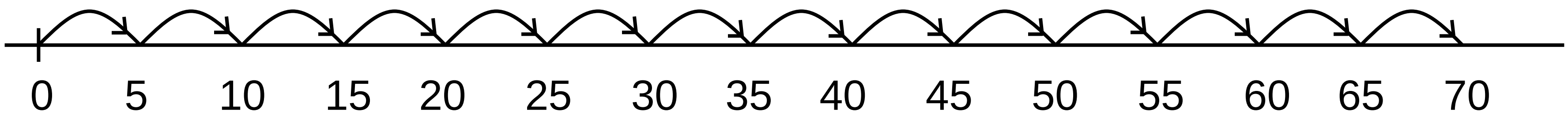
**The number line helps me see each group of 5 clearly.
If I add 5 fourteen times, that is the same as 5 multiplied by 14 (5 x 14). I can make 14 individual jumps of 5 along the number line, or 1 jump of 5 x 10 and 1 jump of 5 x 4. Table facts will help me do this more quickly.**



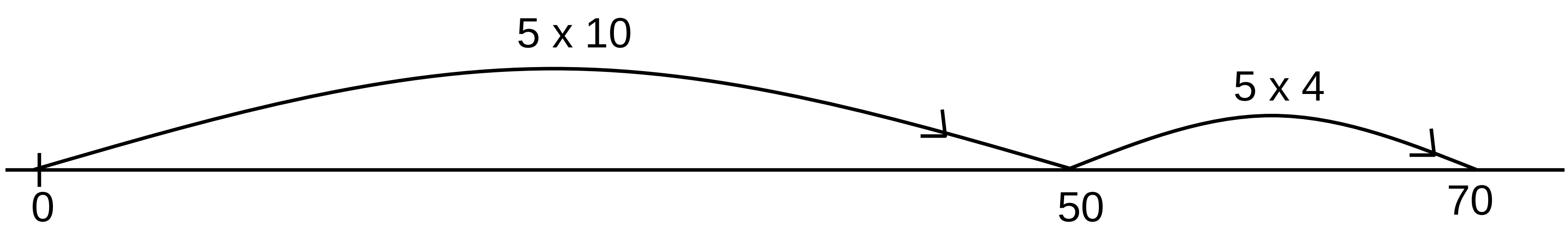
TU x U

14 x 5

The number line shows 5 multiplied by 14. This is equal to 14 multiplied by 5 (14 jumps of 5 on the number line).



Multiplication is *repeated addition.*



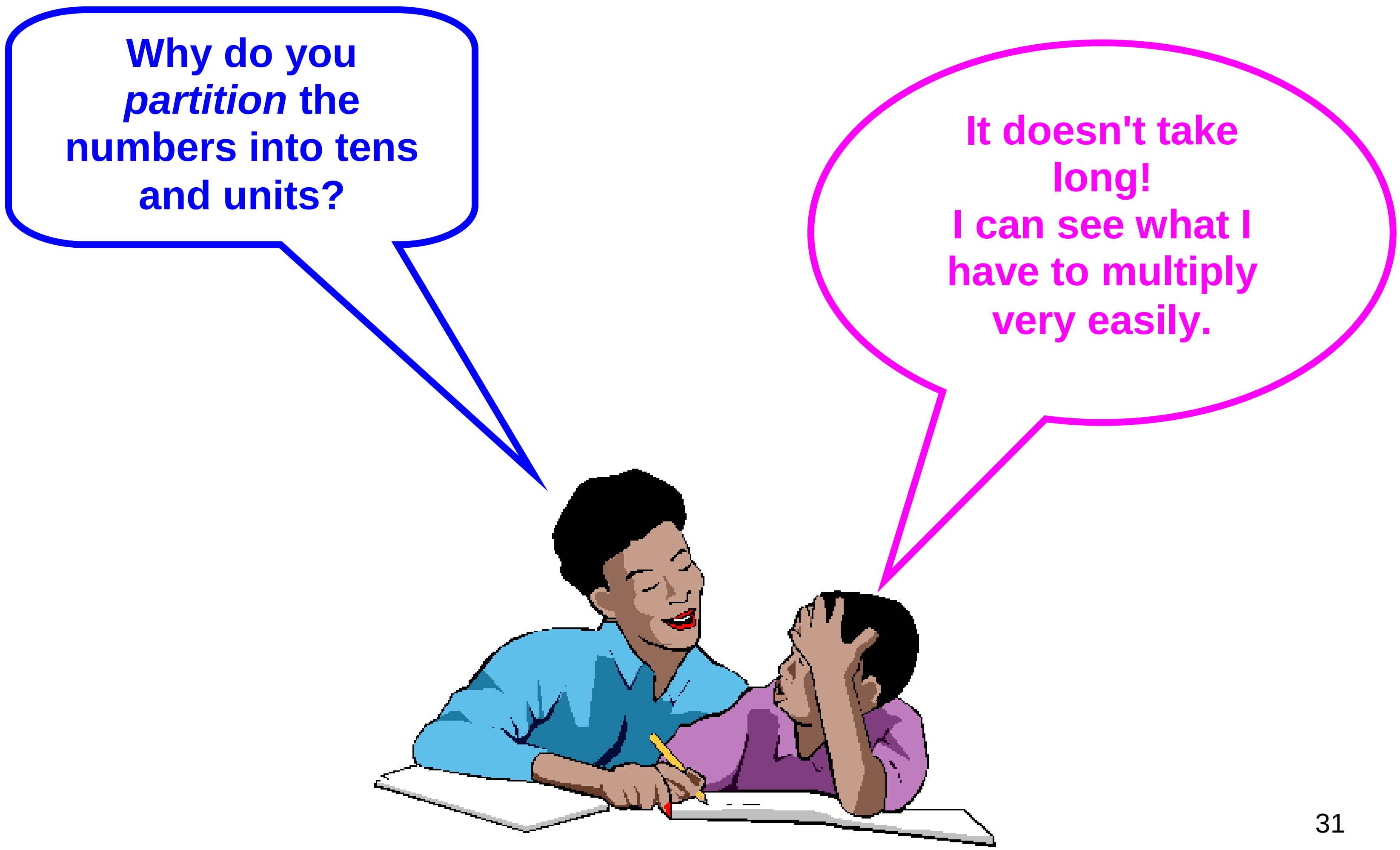
Using table facts to make bigger jumps is more efficient.

14 x 5 = 70

GRID MULTIPLICATION

TU X U

14 x 5



TU X U

14 x 5

Partition TU number into tens and units parts.

14 becomes 10 and 4

14 x 5

X	10	4	
5	50	20	**70**

The *part – products* are totalled to give the *final product* or answer of 70.

50 comes from multiplying 10 by 5. It is called a *part-product.*

20 comes from multiplying 4 by 5. Another *part-product.*

14 x 5 = 70

GRID MULTIPLICATION

TU X TU

46 x 32

Isn't it difficult to multiply 40 by 30?

I know that 30 is 3 x 10 and multiplying by 10 is easy so I do 40 x 3 x 10 = 120 x 10 = 1200.

You've got to do a lot of calculations – don't you get confused?

The layout of the grid helps me organise what I have to do. I like this method.

TU X TU

46 x 32

Both numbers are *partitioned* into their tens and units parts, 46 becomes 40 and 6 and 32 becomes 30 and 2.

46 x 32

X	40	6	
30	1200	180	1380
2	80	12	92
			1472

The *part products* are added in stages to give the final *product* or answer of 1472.

46 x 32 = 1472

MULTIPLICATION

Grid method, Expanded method and Compact method

TU X U

23 x 8

What are the brackets for in the expanded method?

They remind me which numbers I am multiplying.
I also have to remember to line the numbers up as hundreds, tens and units.

Why do you multiply 3 by 8 first in the compact method?
In all the other methods I've noticed that you've multiplied the tens number first!

I multiply the units first so I can carry forward any tens I need to!
This method is very quick but I have to remember to add on any numbers I carry forward.

TU X U

23 x 8

GRID METHOD

X	20	3	
8	160	24	184

EXPANDED METHOD

20 multiplied by 8 equals 160.
3 multiplied by 8 equals 24.

```
 HTU
  23
 x 8
 ---
 160  (20 x 8)
  24   (3 x 8)
 ---
 184
```

Final product from totalling the *part-products*.

COMPACT METHOD
(short multiplication)

```
 HTU
  23
 x 8
 ---
 184
 ---
  2
```

3 multiplied by 8 equals 24 (the first *part product*).

2 is the 2 tens that need to be carried forward and added to the next *part product*.

20 multiplied by 8 equals 160 (2nd *part product*), **plus** the 2 tens equals 180.

The digits are put in the correct columns, to give the answer 184.

23 x 8 = 184

MULTIPLICATION

Grid method, Expanded method and Compact method

TU X TU

46 x 32

I recognise the long multiplication method. How do you multiply 46 by 30?

Well!… I know that 46 x 30 is the same as 46 x 3 x 10. I know my answer will end in zero when I multiply this whole number by 10. So… I put the zero in first. Then I multiply 46 x 3 using the short multiplication method.

TU X TU

46 x 32

GRID METHOD

X	40	6	
30	1200	180	1380
2	80	12	92
			1472

COMPACT METHOD
(long multiplication)

```
   46
 X 32
   92
   1
  380
  1 1
 1472
```

1. First multiply the lower unit by the upper unit (2x6).
2. Next multiply the lower unit by the upper ten (2x4) where the 4 represents 4 X 10 = 40
3. Then start a new row with a 0 as a place holder as you are multiplying by a ten.
4. Multiply the lower ten by the upper unit (3x6)
5. Next multiply the lower ten by the upper ten (3x4)
6. Finally add the two rows to get to your final answer.

46 x 32 = 1472

EXPANDED METHOD

The 4 *part products* are set out vertically underneath the calculation.

```
   46
 x 32
 1200   (40 x 30)
  180    (6 x 30)
   80    (40 x 2)
   12     (6 x 2)
 1472
```

Part products totalled to give final product.

HEALTH WARNING!!!

Children should not be moved on to this method until their understanding of place value is secure.

Division ÷

- divided by
- share
- divide
- share equally
- divisible by
- divide into
- group

DIVISION

Introducing division from multiplication

TU ÷ U

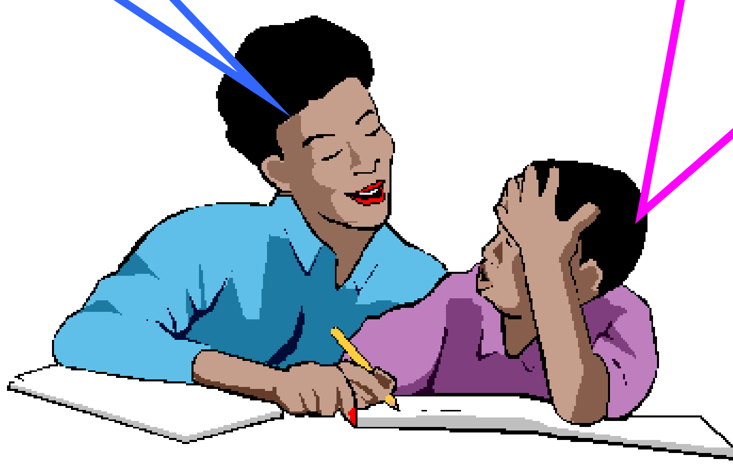
48 ÷ 8

I don't understand how multiplication is related to division.

Well multiplication and division are the inverse of each other.

If I know . . .
6 X 8 = 48

then . . .
48 ÷ 8 = 6
And 48 ÷ 6 = 8



DIVISION

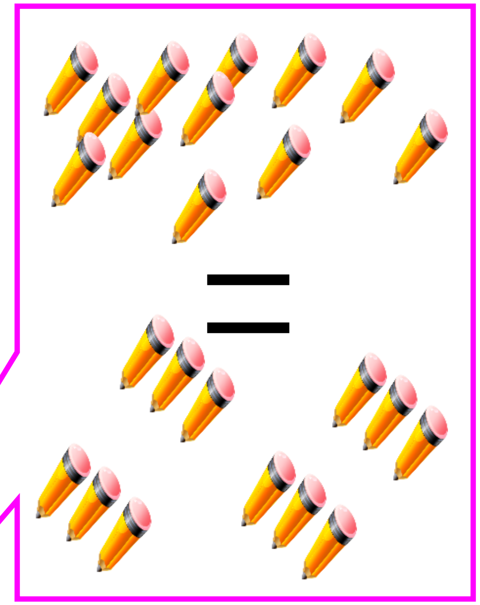
TU ÷ U

12 ÷ 3

Dividing by putting real objects into groups.

Why are you using pencil to do this problem?

It helps me understand the problem. Let's say there were 12 pencils and 4 children. I shared the pencils amongst the children.



There are 4 per child!

DIVISION

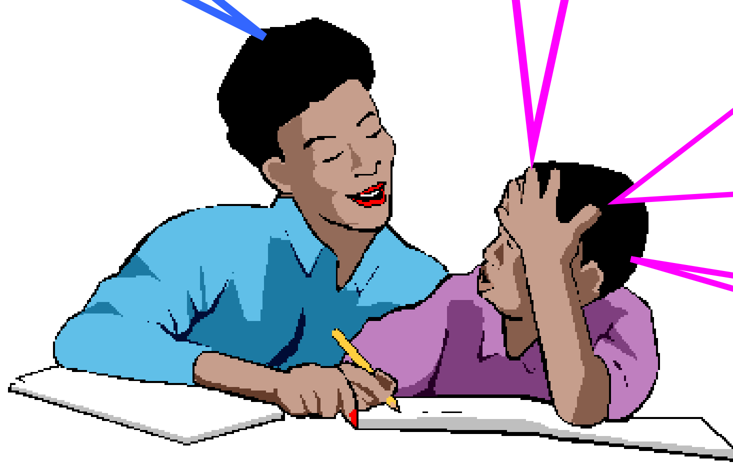
Introducing division by grouping.

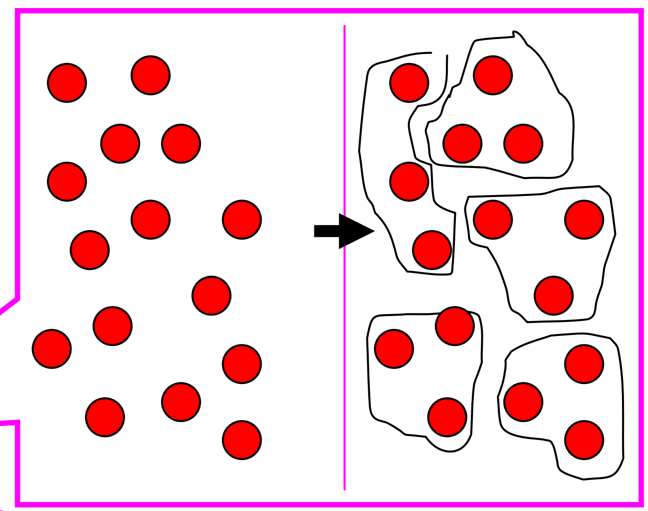
TU ÷ U

15 ÷ 3

Why are you drawing the objects?

I need to see how many groups of 3 there are in 15. I can use pictures or dots to help me.





There's 5 groups!

DIVISION

Introducing division on a number line

TU ÷ U

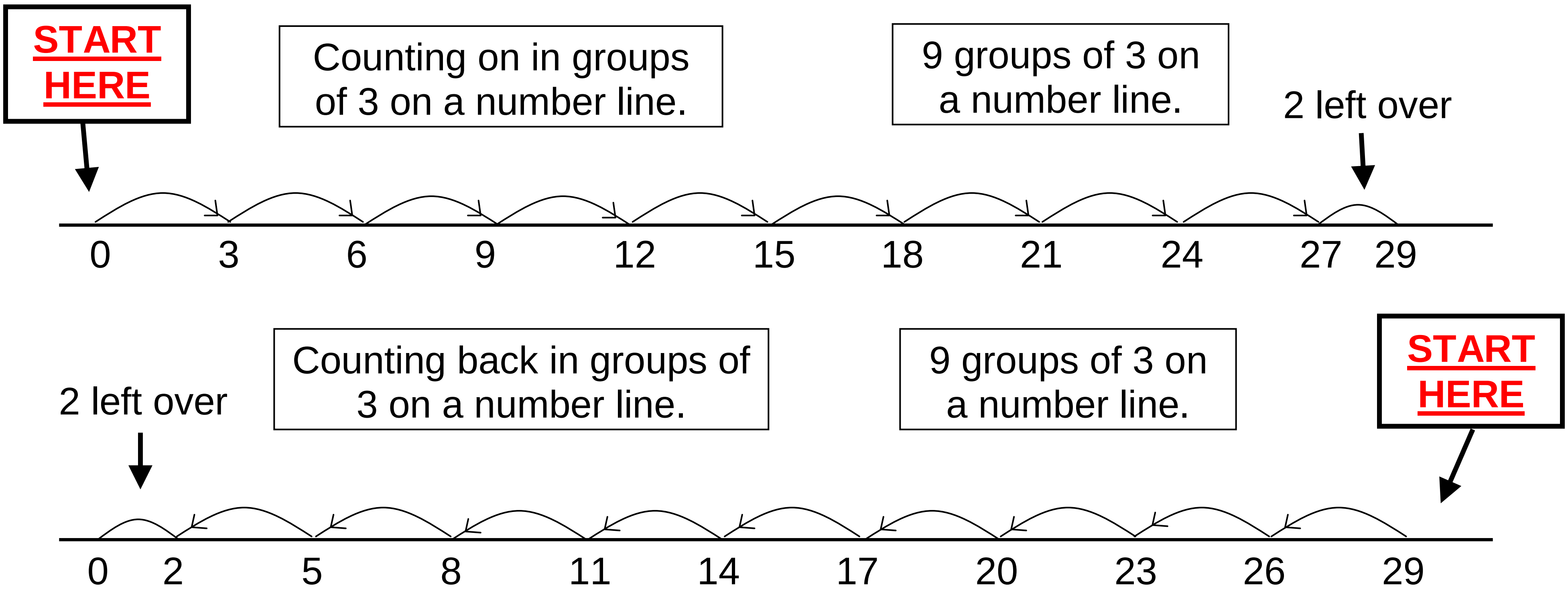
29 ÷ 3

Why are you adding on one line and subtracting on the other? And what has subtraction got to do with division?

I need to see how many groups of 3 there are in 29, so I either add on or take away groups of 3 until I can't add or take any more. Using the subtraction method will help me later on.

TU ÷ U

29 ÷ 3



There are 9 groups of 3 in 29, with 2 left over.

29 ÷ 3 = 9 r2

DIVISION

Chunking on a number line

TU ÷ U

72 ÷ 5

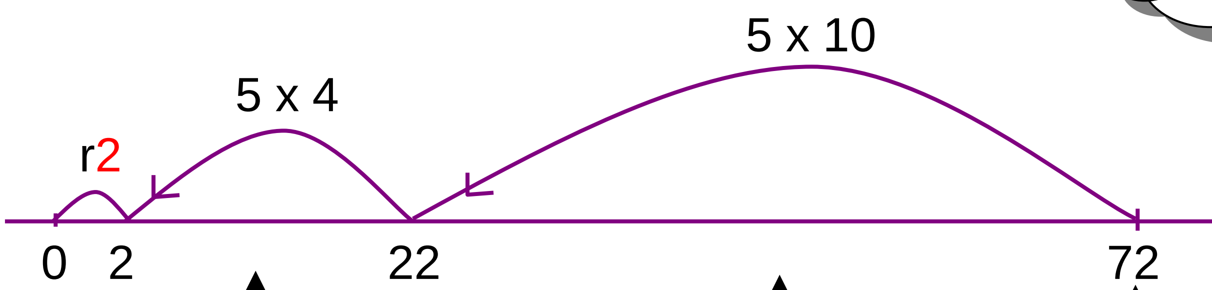
I’ve never heard of chunking before! How does this help with division?

If I can, I try to take out 10 groups of the number I’m dividing by. This is a big chunk and makes the calculation easier. But I can take out chunks that are any number of groups.

TU ÷ U

72 ÷ 5

Numberlines can be vertical or horizontal.



START HERE

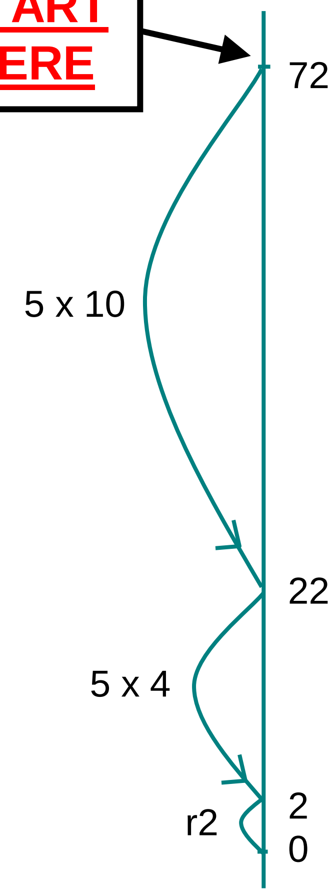
Subtract 4 groups of 5 (20) from 22 to land on 2.

Subtract 10 groups of 5 (50) from 72 to land on 22.

14 groups of 5 subtracted altogether.

2 left!
This is the *remainder*.

START HERE



72 ÷ 5 = 14 r2

DIVISION BY CHUNKING

HTU ÷ U

256 ÷ 7

How do you decide what size chunk to subtract?

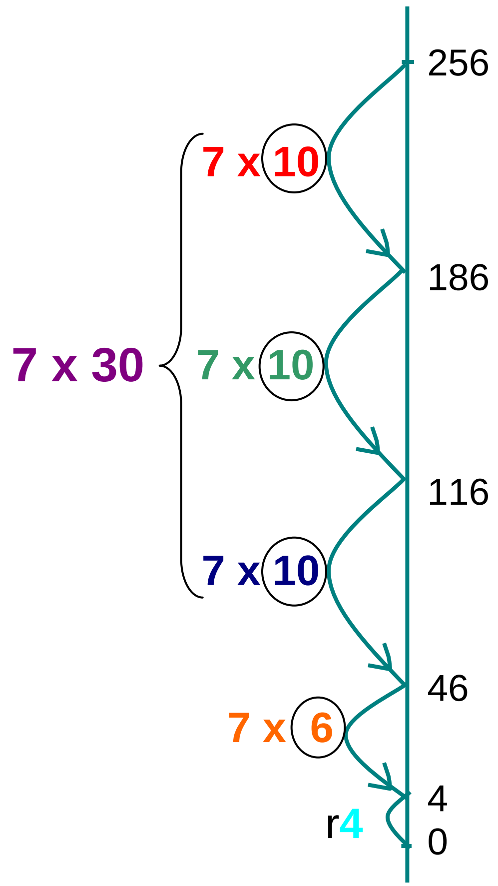
I look for chunks of 10 first. If I take bigger chunks it makes the calculation quicker and easier. Method Ⓒ is shorter and more efficient than Ⓑ .

HTU ÷ U

256 ÷ 7

How many groups of 7 in 256?

A



B

```
 256
 -70  (7 x 10)
 186
 -70  (7 x 10)
 116
 -70  (7 x 10)
  46
 -42  (7 x 6)
   4
```

Subtract chunks of 70 (7 x 10).

How many groups of 7 in 46?

Total the numbers of groups of 7.

10 + 10 + 10 + 6 = 36

C

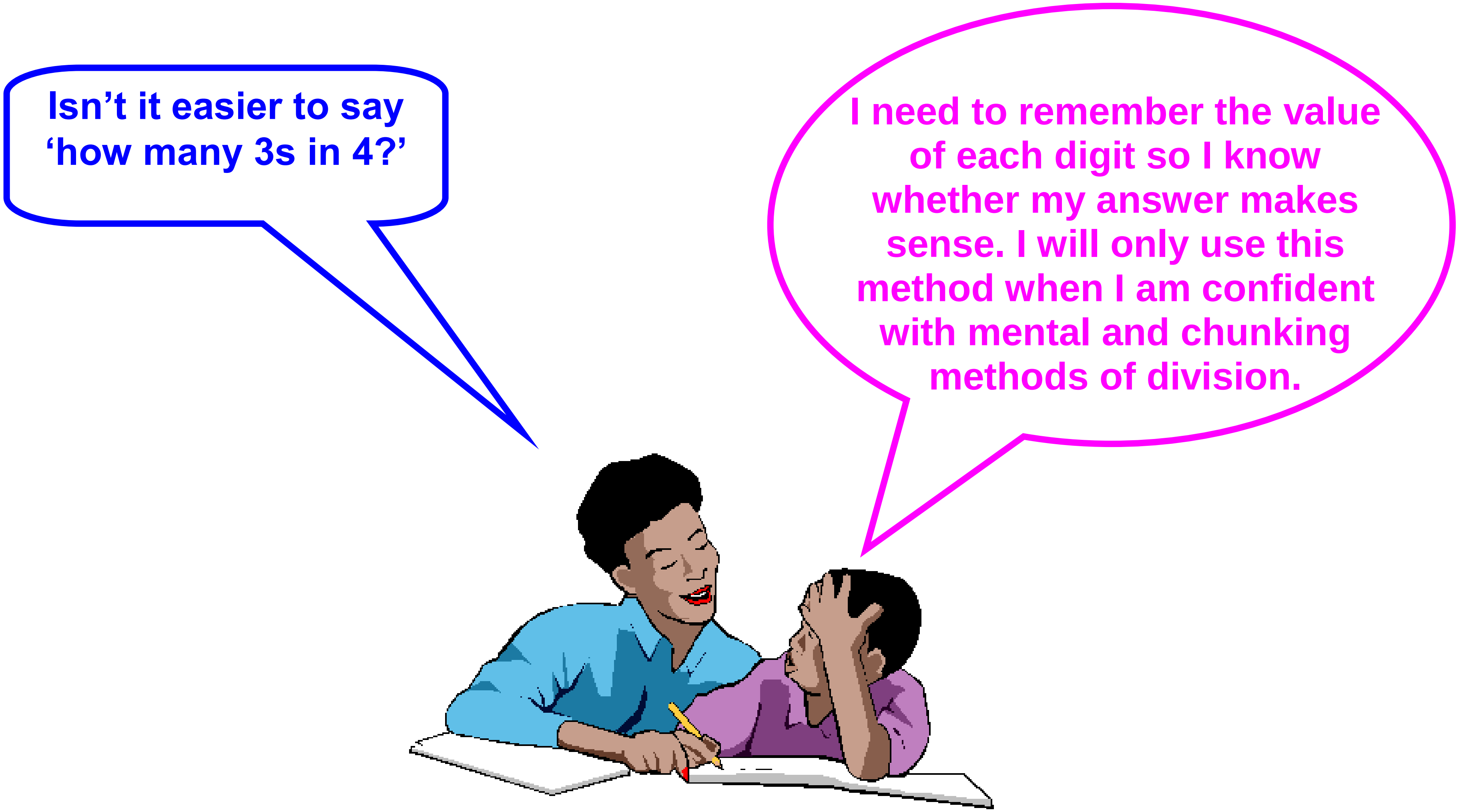
```
 256
-210  (7 x 30)
  46
 -42  (7 x 6)
   4
```

Subtract one large chunk of 210 (7 x 30).

36 groups of 7 have been subtracted and there is 4 left over.

256 ÷ 7 = 36 r4

SHORT COMPACT DIVISION



HTU ÷ U

471 ÷ 3

```
   1
  ____
3| 4¹7 1
```

Q: What is the largest number of hundreds that will divide exactly by 3?
A: 300 divided by 3 = 100. This leaves 100 which is exchanged for ten tens in the tens column.

```
   1 5
  ______
3| 4¹7²1
```

Q: What is the largest number of tens that will divide exactly by 3?
A: 150 divided by 3 = 50. This leaves 20 which is exchanged for 20 units in the units column.

```
   1 5 7
  ______
3| 4¹7²1
```

Q: What is the largest number of units that will divide exactly by 3?
A: 21 divided by 3 = 7

471 ÷ 3 = 157

CALCULATIONS IN CONTEXT

All the methods in this booklet support children in using their mental and written skills to solve calculations. Children need to be encouraged to use the method that they understand and can use confidently.

It is important that children are able to choose the most appropriate method for the calculation. For example:

4003 - 3998

These numbers are very close together and so counting up on a number line (actual or imagined) would be the most efficient method.

200 ÷ 4

Dividing by 4 is the same as halving and halving again. As it is easy to halve 200 and easy to halve 100, this would be the most efficient method.

Using and applying appropriate skills is very important, when calculations are needed to solve a problem.

4 C.DS at £2.99 – how much altogether?

£2.99 is almost £3.00 and so round up, multiply, then adjust:

4 x £3.00 = £12.00

£12.00 – 4p = £11.96

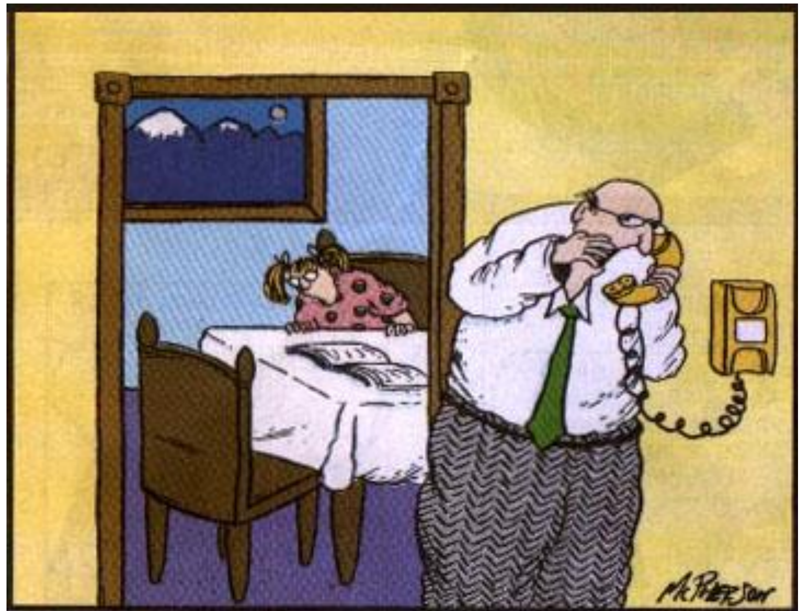
GLOSSARY

These are definitions for use with this booklet. There may be other applications for these words, not covered in this booklet.

Arrays	An arrangement of objects, e.g. counters, into columns and rows.
Chunking	In division, chunking is where multiples of the divisor (the number you are dividing by) are repeatedly subtracted. Initially, children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. 97 ÷ 9 6)196 − 60 6 × 10 136 − 60 6 × 10 76 − 60 6 × 10 16 − 12 6 × 2 4 32 Answer: 32 R 4 In the above example, ‘chunks’ of 6 are being subtracted, initially in ‘chunks’ of 10.
Compact method	Leads on from an expanded method. A compact method is tidier, quicker, not partitioned and starts with the least significant digit. 217 + 147 364 1

Decomposition	A vertical method of subtraction. This method involves taking an amount from a higher place value and giving it to a lower place value, in the top number. The top number is then re-written so that the overall value of the number does not change. 700 + 40 + 6 is equal to 700 + 30 + 16. The number in the top line is broken down to aid calculation. Example: For 719 – 297 the calculation is written as $_6$~~7~~ $_1$1 9 – 2 9 7 4 2 2
Expanded method	A move to a layout showing the addition of the hundreds to the hundreds, the tens to the tens and the ones to the ones, separately. This will lead on to a compact method. 700 + 50 + 6 + 200 + 20 + 2 900 + 70 + 8 113 + 147 10 50 200 260
Informal written methods	Pencil and paper methods for supporting, recording or explaining calculations. When multiplying, this might be the grid method.
Inverse	The inverse operation of addition is subtraction (visa versa). The inverse of multiplication is division (visa versa).

Multiples	The result of multiplying by a whole number. Examples: 4 × 5 = 20 20 is a multiple of 4 and also of 5. Some multiples of 2 are: 2,6,8, 200, 360…
Partition	To split a number into component parts. Example: the two-digit number 38 can be partitioned into 30 + 8.
Product	The result of multiplying. For example: The product of 3 and 6 is 18.
Significant digits	The digit that has the highest value. E.g. in 24, the 2 is the most significant digit as it is 2 tens, whereas 4 is 4 ones or 4 units.
Standard written method	This is where an efficient written method has been developed, which is applied universally. It could be partitioned or expanded. 217 700 + 50 + 6 + 147 + 200 + 20 + 2 364 900 + 70 + 8 1



"Uh, yeah, Homework Help Line? I need to have you explain the quadratic equation in roughly the amount of time it takes to get a cup of coffee."

"Y'know, you're right! This homework isn't bad!"

J Nicholls and T Benyon 09/03/2018
(Adapted from 'They Didn't Do it Like That in My Day' by Brighton and Hove Numeracy Team)